**Involutive Substructural Logics**
- Provide adequate languages to reason in presence of vague or incomplete information about resources, dynamic data structures and algebraic varieties.
- Obtained by extending Multiplicative Additive Linear Logic (MALL) [2] with Hilbert Axioms.

**Analytic Calculi**
Calculi where all derivations proceed by step-wise decomposition of the formula to be proved are called analytic. Analytic calculi are:
- Prerequisite for developing automated reasoning methods.
- Key to establish essential properties of the formalized logics.

**Why Systematic Proof Theory?**
Introducing analytic calculi is often laborious and error prone. Having automated procedures for this purpose is very desirable. The systematic procedure in [1] transforms Hilbert axioms in the language of MALL into equivalent analytic rules in sequent and hypersequent calculus.

**Background**

### Multiplicative Additive Linear Logic MALL
The language of MALL consists of propositional variables \( \mathcal{V} = \{ a, b, c, \ldots \} \), their duals \( \mathcal{V}^⊥ = \{ a^⊥, b^⊥, c^⊥, \ldots \} \), the constants \( \{ ⊥, T, 1, 0 \} \), and the logical connectives \( \{ \&c, ⊗, □ \} \). The formulas in MALL are generated by:

\[
\mathcal{F} ::= \mathcal{V} | \mathcal{V}^⊥ | T | 1 | 0 | \mathcal{F} \& \mathcal{F} | \mathcal{F} \& \mathcal{F} | \mathcal{F} \oplus \mathcal{F} | \mathcal{F} \oplus \mathcal{F} | 1 | 0
\]

### Logical Connectives
- The two conjunctions \( □ \) (times) and \&c (with) represent the availability of two actions. In case of □ both actions will be performed. In case of &c we can chose to perform either of the actions, but not both.
- The two disjunctions \( ∨ \) (plus) and \( \oplus \) (the dual of &c) represents a non-deterministic choice of one action. \( ∨ \) (the dual of □) represents the dependency between actions.
- The linear negation \( (\cdot)^⊥ = \text{involution} \), i.e. \( A^⊥⊥ \Leftrightarrow A \).

### The Hypersequent System HMLL

#### Involutive Substructural Logics

**From Axioms to Analytic Rules - The Procedure**
- Transforms axioms within classes \( N^2 \) and \( P^2 \) into a set of analytic rules.
- Uses the (hyper)sequent calculus for MALL (HMALL) as base calculus for axioms within \( N^2 \) (\( P^2 \)).
- The procedure consist of the following steps:
  1. Transforms axioms into equivalent (hyper)structural rules, i.e. rules with no occurrence of logical connectives.
  2. Verifies the acyclicity condition - if the (hyper)structural rule is cyclic applies weakening to obtain an acyclic rule.
  3. Applies rule completion to transform the acyclic rules to analytic.

**Example**
The axiom \( \text{Inv}: (A ⊥ &B)_{k1} \oplus (B ⊥ &A)_{k1} \in \mathcal{P}_3 \) is transformed into the analytic rule (com) through these steps:

\[
\Gamma \vdash (A ⊥ &B)_{k1} \oplus (B ⊥ &A)_{k1} \Rightarrow \Gamma \vdash A ⊥ &B \Rightarrow \Gamma \vdash B ⊥ &A \Rightarrow \Gamma \vdash A ⊥, B \Rightarrow \Gamma \vdash B ⊥, A \Rightarrow \Gamma \vdash \theta, A \Rightarrow \Gamma \vdash \sigma, B \Rightarrow \Gamma \vdash \theta, \sigma \Rightarrow \Gamma \vdash \theta, \sigma
\]

Figure: Transformation steps for linearity (com)

**This Thesis**
In this thesis we developed InvAxiomCalc, which provides an implementation in Prolog of the systematic procedure in [1] for axioms in the classes \( N^2 \) and \( P^2 \).

### Implementation Details
The implementation consists of:
1. Identifying the class in Substructural Hierarchy of an axiom.
2. Transforming the axiom within class \( N^2 \) into a set of (hyper)structural rule.
3. Transforming, if necessary, the (hyper)structural rule to acyclic.
4. Applying the completion procedure to the rule from step (3).
5. Generating a \( \text{LaTeX} \) paper with the transformation results.

**InvAxiomCalc as Command Line Tool**
- InvAxiomCalc can be executed in a Prolog environment.
- Prints the generated results in the terminal.
- Generates a \( \text{LaTeX} \) paper with the transformation steps.

**InvAxiomCalc on the Web**
- A web interface for running InvAxiomCalc is available at https://logic.at/tinc/webinvaxiomcalc/.
- Displays the output printed by the terminal.
- Generates a paper with the transformation result.

### References